

Information Theory and Coding (CM303)

Midterm (40%)

28 May 2018

Instructor: Eng. Hosam Almqadim

Time Allowed: 2 hours

Q1. (10 Marks) Consider the following information channel, the channel input **A** with symbols **a1, a2, and a3**, and probabilities $p(a1)=0.6$, $p(a2)=0.3$, and $p(a3)=0.1$. The channel output **B** with symbols **b1, b2, and b3**. The channel is fully specified by the following channel matrix

$$p(B/A) = \begin{bmatrix} ? & 0.5 & 0 \\ ? & 0 & 0.5 \\ 0 & 0.5 & ? \end{bmatrix}$$

- a) Find the **missing probabilities** and **draw the schematic** of the channel? (2 Marks)
- b) Obtain the output probabilities $p(b1)$, $p(b2)$ and $p(b3)$? (2 Marks)
- c) Find the Mutual Information $I(A;B)$ and **Channel capacity C**? (4 Marks)
- d) Find the **probabilities** and **entropy** of the **2nd order extension Z(A²)**? (2 Marks)

Q2. (04 Marks) In a telegraph source having **two independent symbols** dot and dash, the **dot duration** is **1/3** of the **dash duration** and the total duration is **0.8 sec**. The probability of **dash occurrence** is **one third of dot occurrence** and the **time separation between symbols** is **0.1 sec**. Find the following:

- A. The **information rate** of this telegraph. (2 Marks)
- B. The **maximum possible information rate** with the same average symbol duration. (2 Marks)

Q3. (14 Marks) Consider a Discrete Memoryless Source with symbols $S = \{a,b,c,d,e,f,g\}$ have probabilities $P(S) = \{3/14, 1/14, 1/7, 1/7, 2/7, 1/14, 1/14\}$ respectively.

- 1. Construct a **Huffman code** for the source? (2 Marks)
- 2. Compute the **average code length**, the **efficiency** and **redundancy** of the code? (3 Marks)
- 3. Is this code an **optimal code**? (Justify your answer) (2 Marks)
- 4. Find the **Information Rate** if the DMS generates 1 symbol randomly every 1µsec? (2 Marks)
- 5. **Encode** the following message: **aabacaafggefad** (2 Marks)
- 6. Draw the **decision tree** of the code. (1 Marks)
- 7. **Decode** the following bit stream: **01010001101100000011011110000001...** (2 Marks)

Q4. (12 Marks) Given the following code

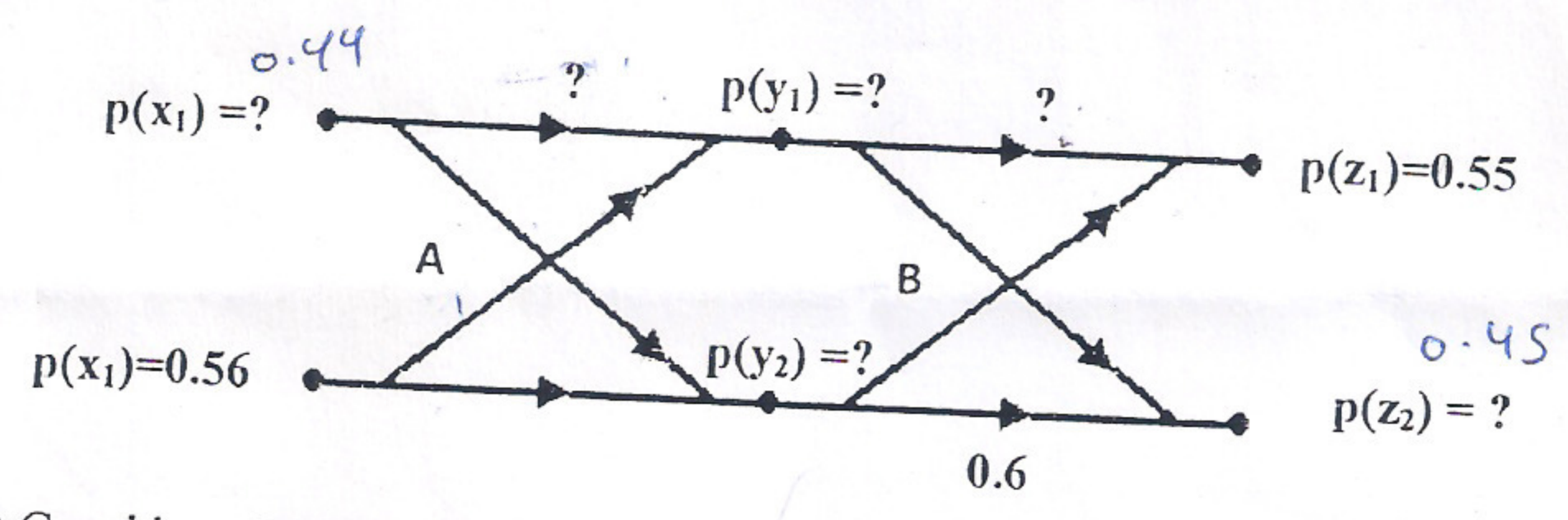
message	Code Word	message	Code Word	message	Code Word	message	Code Word
0000	000 0000	0100	110 0100	1000	101 1000	1100	011 1100
0001	110 0001	0101	000 0101	1001	011 1001	1101	101 1101
0010	011 0010	0110	101 0110	1010	110 1010	1110	000 1110
0011	101 0011	0111	011 0111	1011	000 1011	1111	110 1111

- a) Find the **Generator matrix** of the code? (4 Marks)
- b) Find the **error detection capability** and **error correction capability**? (2 Marks)
- c) Find the **parity check matrix** of the code and its **Transpose**? (2 Marks)
- d) Is this code a **linear code**? (Justify your answer) (2 Marks)
- e) **Encode** the following bit stream **10110010000110111100.....** (2 Marks)

$\frac{\text{bit}}{\text{sec}} \cdot \frac{\text{Sym}}{\text{sec}}$

Good luck

Q1. (12 Marks) For the two Binary Symmetrical Channels (A & B) connected in cascade as shown below, Find the input, output and channels forward probabilities.



Q2. (8 Marks) Consider a DMS Source with symbols $S_i, i=1,2,3,4$. Table below lists 6 possible binary codes

Table of Codes of the source S						
S_i	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

- a) Find which of them **distinct** codes are? C_2, C_3, C_4, C_5, C_6 (2 Marks)
- b) Find which of them **prefix-free** codes are? C_1, C_4, C_6 (2 Marks)
- c) Find weather **instantaneous** codes are existence for these codes? (2 Marks)
- d) Can you **decide** which code is the **best** code for this source, and why? (2 Marks)

Q3. (24 Marks, 3 each) Consider the Systematic Linear Block Code with the following syndrome look-up table.

Error Pattern (e)	Syndrome (S)
0000000	000
0000001	110
0000010	011
0000100	101
0001000	111
0010000	001
0100000	010
1000000	100

- a) Find the **Generator matrix** of the code? I
- b) Find all the **Code Words** and the **Minimum Hamming Distance**? $010, 001$
- c) Find code bits, message bits, parity bits, code rate, the **error-detection and error-correction capabilities** of the code?
- d) Write down the **Parity Check Equations** and draw the **Encoder**? $11, 10, 01, 110$
- e) Are the generator vectors **linearly independent**? (Justify your answer)
- f) Is this code a **linear code**? (Justify your answer)
- g) Encode the bit stream, $m = 11011110001001\dots$? $00, 01, 110$
- h) If $r_1 = 1101011$ and $r_2 = 0101101$ were received, what are the transmitted code-words and original messages? (clearly show the recovery steps)

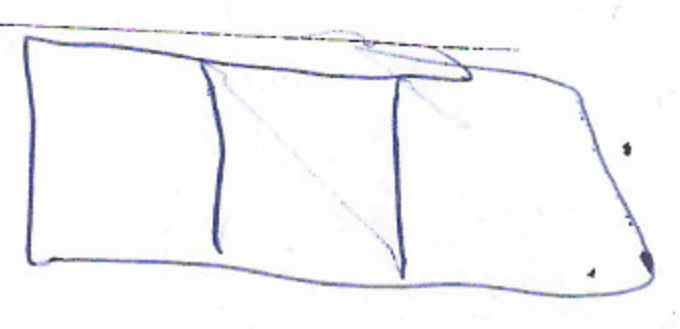
Q4. (16 Marks) Given a Binary Convolutional Encoder with $K=3$, rate $1/3$, and Impulse Response

101011010.

- a) **Encode** the input sequence bits $m = 10101$? $n=9$
- b) **Draw** the encoder? (show your answer) 101
- c) **Draw** the **Trills diagram** of the encoder? $100, 101, 010, 011, 000, 001, 100, 101$

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 لحسام الدين الهنشيرى

$S = eH$
 $S = rHT$



$P_{n-k \times k}$



Information Theory and Coding (CM303)

Midterm (40%)

26 Dec 2017

Instructor: Eng. Hosam Almqadim

Time Allowed: 2 hours

Q1. (12 Marks) Consider the following information channel, the channel input A with symbols $a_1, a_2,$ and a_3 , and probabilities $p(a_1)=0.6, p(a_2)=0.3,$ and $p(a_3)=0.1$. The channel output B with symbols $b_1, b_2,$ and b_3 . The channel is fully specified by the following channel matrix

$$p(B/A) = \begin{matrix} & \begin{matrix} b_1 & b_2 & b_3 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \end{matrix}$$

- a) Draw the schematic of the channel? (2 Marks)
- b) Obtain the output probabilities $p(b_1), p(b_2)$ and $p(b_3)$? (3 Marks)
- c) Find the Mutual Information $I(A;B)$? (5 Marks)
- d) Find the probabilities and entropy of the 2nd order extension Z of A ? (2 Marks)

$\rightarrow H^2(S) = 2 \times H(S)$

$R_S H(S) \rightarrow \frac{1}{64} \times \frac{1}{64}$

Q2. (10 Marks) In a telegraph source having two independent symbols dot and dash, the dot duration is 0.2 sec and the dash duration is 0.6 sec. The probability of dot occurrence is three times that of dash and the time separation between symbols is 0.1 sec. Find the following:

- A. The information rate of this telegraph. (5 Marks)
- B. The maximum possible information rate with the same average symbol duration. (5 Marks)

Q3. (18 Marks) Consider a Discrete Memoryless Source with symbols $S = \{a,b,c,d,e,f,g\}$ have probabilities $P(S) = \{2/7, 3/14, 1/7, 1/7, 1/14, 1/14, 1/14\}$ respectively.

- 1. Construct a Huffman code for the source? (4 Marks)
- 2. Compute the average code length of the code. (2 Marks)
- 3. Find the efficiency and the redundancy of the code? (2 Marks)
- 4. Is this code an optimal code? (Justify your answer) (2 Marks)
- 5. Find RI if the DMS generates 1 symbol randomly every 1 μ sec? (2 Marks)
- 6. Encode the following message: aabacaafggfad (2 Marks)
- 7. Draw the decision tree of the code. (2 Marks)
- 8. Decode the following bit stream: 0101000110110010000110111100000001.... (2 Marks)

a a g e d d c e b d f a

dot = 3dash

Good luck

Information Theory and Coding (CM303)

Midterm II (35%)

13 May 2017

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1. (9 Marks) Consider the following information channel, the channel input A with symbols $a_1, a_2,$ and $a_3,$ and probabilities $p(a_1)=0.6, p(a_2)=0.3,$ and $p(a_3)=0.1.$ The channel output B with symbols $b_1, b_2,$ and $b_3.$ The channel is fully specified by the following channel matrix

$$p(B/A) = \begin{matrix} & \begin{matrix} b_1 & b_2 & b_3 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \end{matrix} \begin{matrix} (b_1/a_1) (b_2/a_1) (b_3/a_1) \\ (b_1/a_2) (b_2/a_2) (b_3/a_2) \\ (b_1/a_3) (b_2/a_3) (b_3/a_3) \end{matrix}$$

- a) Draw the schematic of the channel?
 b) Find the Mutual Information $I(A;B)$?

(3 Marks)
(6 Marks)

Q2. (10 Marks) Consider a source with a six-symbol alphabet, $X_1, X_2, X_3, X_4, X_5,$ and $X_6,$ with probabilities $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02,$ and $P_6 = 0.04,$ respectively.

- A. Find a Shannon-Fano code for this source.
 B. Compute the average code length of the code.
 C. Design a decision tree of the code?
 D. Is the code an optimal code? (justify your answer)

(4 Marks)
(2 Marks)
(2 Marks)
(2 Marks)

Q3. (16 Marks) Given the following code

message	Code Word	message	Code Word
0000	000 0000	1000	101 1000
0001	110 0001	1001	011 1001
0010	011 0010	1010	110 1010
0011	101 0011	1011	000 1011
0100	110 0100	1100	011 1100
0101	000 0101	1101	101 1101
0110	101 0110	1110	000 1110
0111	011 0111	1111	110 1111

- a) Find the Generator matrix of the code?
 b) Find the error detection capability and error correction capability?
 c) Find the parity check matrix of the code and its transpose?
 d) Is this code a linear code? (Justify your answer)
 e) Are the generator vectors linearly independent? (Justify your answer)
 f) Encode the following bit stream 10110010000110111100.....
 g) Draw an encoder of the code?

(4 Marks)
(2 Marks)
(2 Marks)
(2 Marks)
(2 Marks)
(2 Marks)
(2 Marks)

1011 → 000 1011
 0010 → 011 0010

Good luck

u = m G

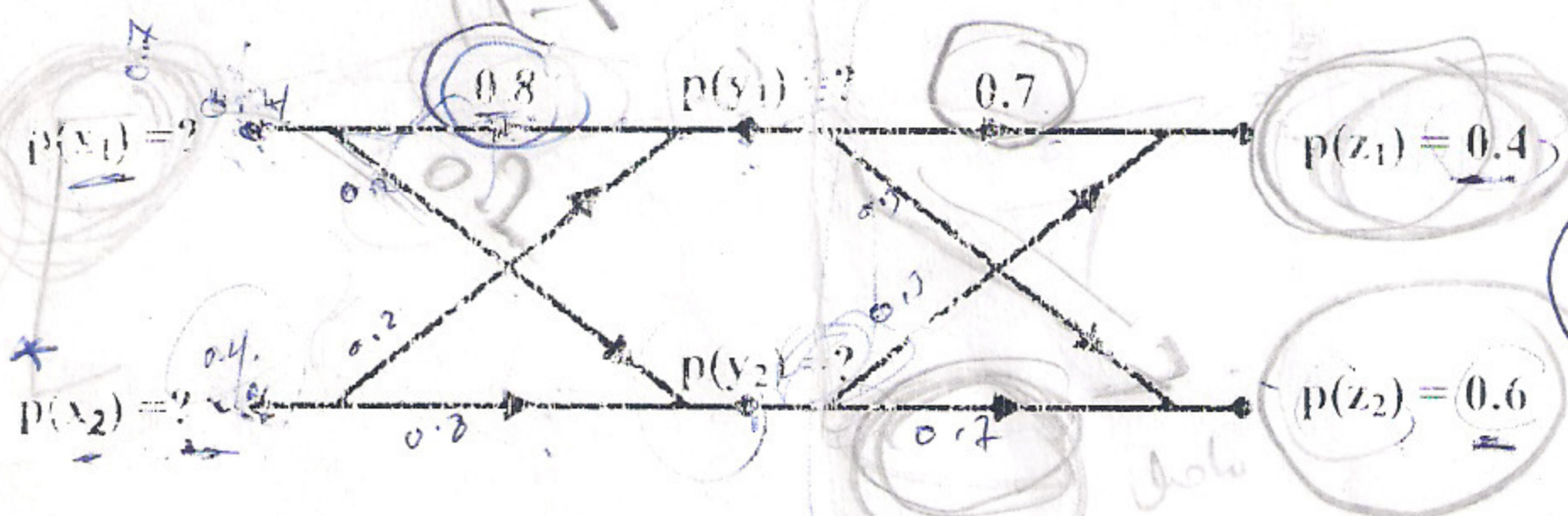
Q1. A discrete source transmits messages x_1, x_2, x_3 with probabilities $p(x_1) = 0.3, p(x_2) = 0.25, p(x_3) = 0.45$. The source is connected to the channel whose conditional probability matrix is

$$P(Y/X) = \begin{matrix} & y_1 & y_2 & y_3 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix} \end{matrix}$$

$$= P(y_j | x_i) P(x_i)$$

- A. Draw the channel schematic? And Obtain the joint probability matrix $P(X, Y)$? (4 Marks)
- B. Obtain the probabilities $p(y_1), p(y_2)$ and $p(y_3)$? (6 Marks)

Q2. Consider two binary symmetrical channels are connected in cascade as shown below:



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الاحتمالات اقل

Find the probabilities $p(x_1), p(x_2), p(y_1)$ and $p(y_2)$? (8 Marks)

Q3. (16 Marks) Consider a source with a six-symbol alphabet, X_1, X_2, X_3, X_4, X_5 , and X_6 , with probabilities $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02$, and $P_6 = 0.04$, respectively.

- 1. Find a Shannon-Fano code for this source.
- 2. Compute the average code length of the code.

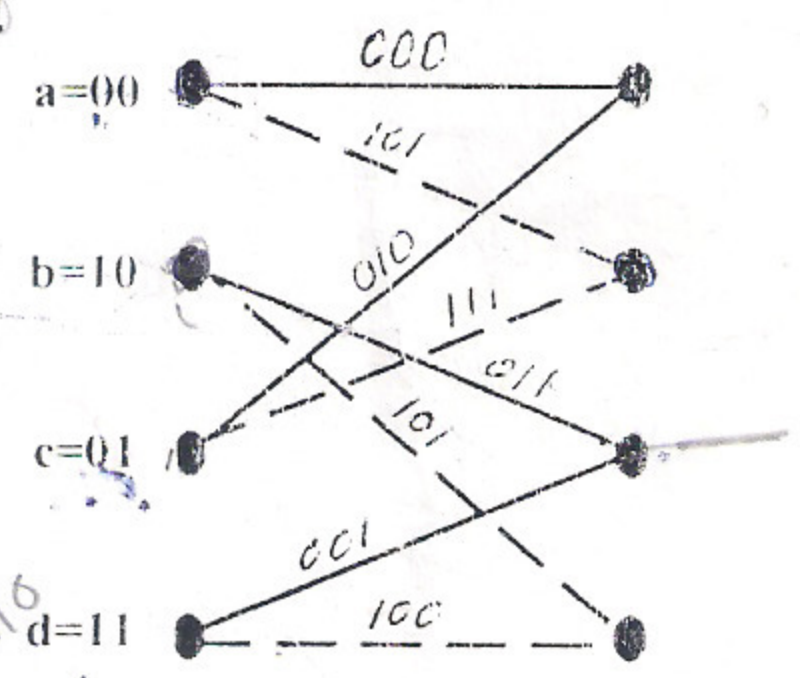
$$t = \left\lfloor \frac{d \cdot m - 1}{2} \right\rfloor = 1$$

Q4. (10 Marks) Consider the Systematic Linear Block Code with the following parity check matrix.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- A. if the code is a single error-correction code, find the syndrome look-up table. (5 Marks)
- B. For the received vectors r_1 and r_2 , recover the transmitted code-word and the original message.
 $r_1 = 1101011, r_2 = 0101101$. (clearly show the recovery steps) (6 Marks)

Q5. (15 Marks) Given the following trellis diagram of a convolutional code.



- A. Encode the input sequence bits $m = 10101$
- B. Decode the received vector r using Viterbi Decoding.
 $R = 101 \ 111 \ 001 \ 000 \ 000$.
- C. How many errors are there in the received vector R (identify the error bits) and show the transmitted vector U

$$e = s - rH^T$$

علم التوافق

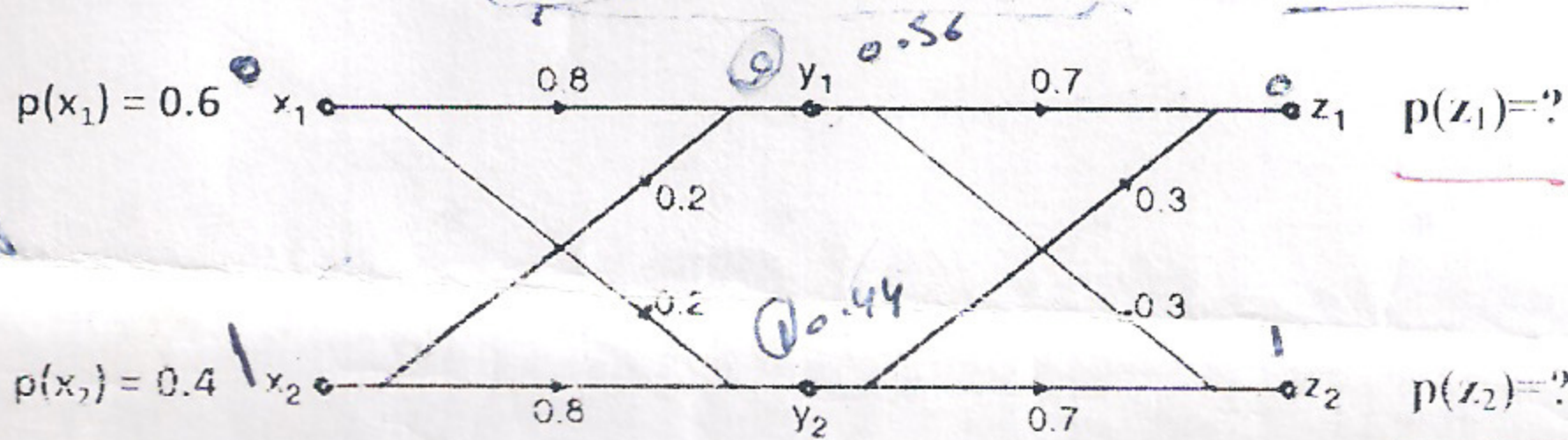
Q1. (16 Marks) In a telegraph source having two independent symbols dot and dash, the dot duration is 0.2 sec and the dash duration is 0.6 sec. The probability of dot occurrence is three times that of dash and the time separation between symbols is 0.1 sec. Find the following:

- A. The information rate of this telegraph. (12 Marks)
- B. The maximum possible information rate with the same average symbol duration. (4 Marks)

Q2. (16 Marks) Consider a source with a six-symbol alphabet, $X_1, X_2, X_3, X_4, X_5,$ and $X_6,$ with probabilities $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02,$ and $P_6 = 0.04$ respectively.

- A. Find a Shannon-Fano code for this source. (8 Marks)
- B. Compute the average code length of this Huffman code. (8 Marks)

Q3. (8 Marks) Consider two binary symmetrical channels are connected in cascade as shown below:



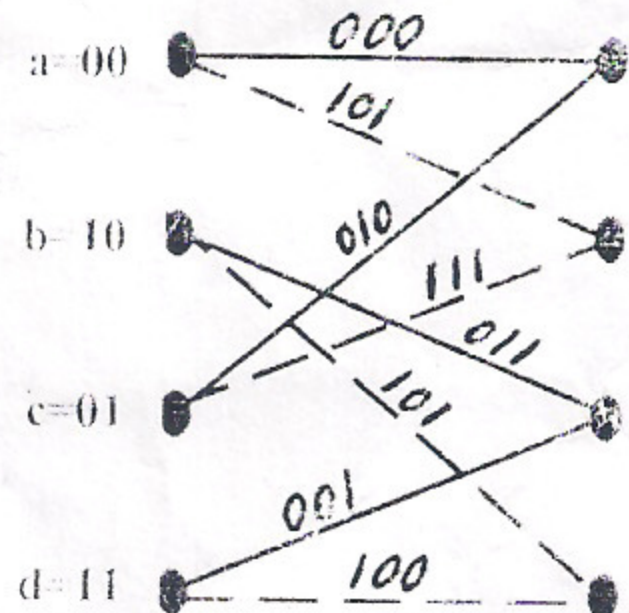
- A. Find the probabilities $p(z_1)$ and $p(z_2)$. (8 Marks)

Q4. (10 Marks) Consider the Systematic Linear Block Code with the following parity check matrix.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- A. If the code is a single error correction code, find the syndrome look-up table. (5 Marks)
- B. For the received vectors r_1 and r_2 , recover the transmitted code-word and the original message
 $r_1 = 1101011, r_2 = 0101101$. (clearly show the recovery steps) (5 Marks)

Q5. (10 Marks) Given the following trellis diagram of a convolutional code.



- A. Decode the received vector r using Viterbi Decoding.
 $R = 101\ 111\ 111\ 100\ 001\ 011$.
- B. How many errors are there in the received vector R
 (identify the error bits) and show the transmitted vector U

$H = [I \mid P^T]$
 $H = \begin{bmatrix} I \\ P \end{bmatrix}$
 $G = [P \mid I]$

bps [RI]

10

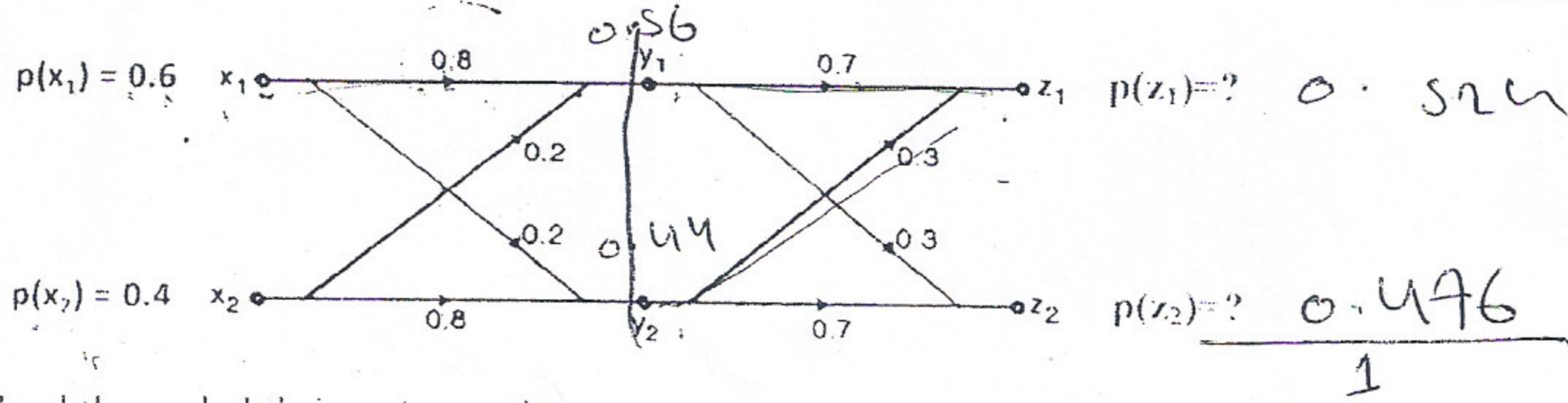
Q1. (16 Marks) In a telegraph source having two independent symbols dot and dash, the dot duration is 0.2 sec and the dash duration is 0.6 sec. The probability of dot occurrence is three times that of dash and the time separation between symbols is 0.1 sec. Find the following:

- A. The information rate of this telegraph. (12 Marks)
- B. The maximum possible information rate with the same average symbol duration. (4 Marks)

Q2. (16 Marks) Consider a source with a six-symbol alphabet. $X_1, X_2, X_3, X_4, X_5,$ and X_6 , with probabilities $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02,$ and $P_6 = 0.04$, respectively.

- A. Find a Shannon-Fano code for this source. (8 Marks)
- B. Compute the average code length of this Huffman code. (8 Marks)

Q3. (8 Marks) Consider two binary symmetrical channels are connected in cascade as shown below



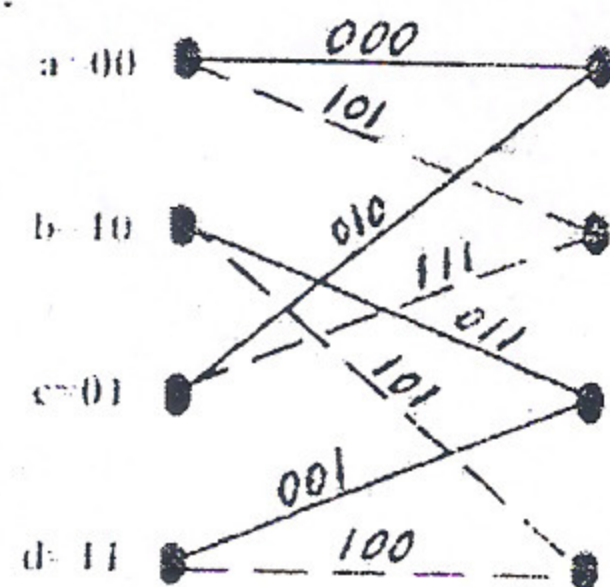
- A. Find the probabilities $p(z_1)$ and $p(z_2)$. (8 Marks)

Q4. (10 Marks) Consider the Systematic Linear Block Code with the following parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- A. If the code is a single error correction code, find the syndrome look-up table. (5 Marks)
- B. For the received vectors r_1 and r_2 , recover the transmitted code-word and the original message. $r_1 = 1101011, r_2 = 0101101$. (clearly show the recovery steps) (5 Marks)

Q5. (10 Marks) Given the following trellis diagram of a convolutional code.



- A. Decode the received vector r using Viterbi Decoding. $R = 10111111000011$.
- B. How many errors are there in the received vector R (identify the error bits) and show the transmitted vector U

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Information Theory and Coding (CM303)

Midterm II (40%)

14 May 2016

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1: (5 points) Consider a DMS S with symbols $S_i, i = 1, 2, 3, 4$. Table below lists 6 possible binary codes

S_i	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

- Find which of them **distinct** codes are?
- Find which of them **prefix-free** codes are?

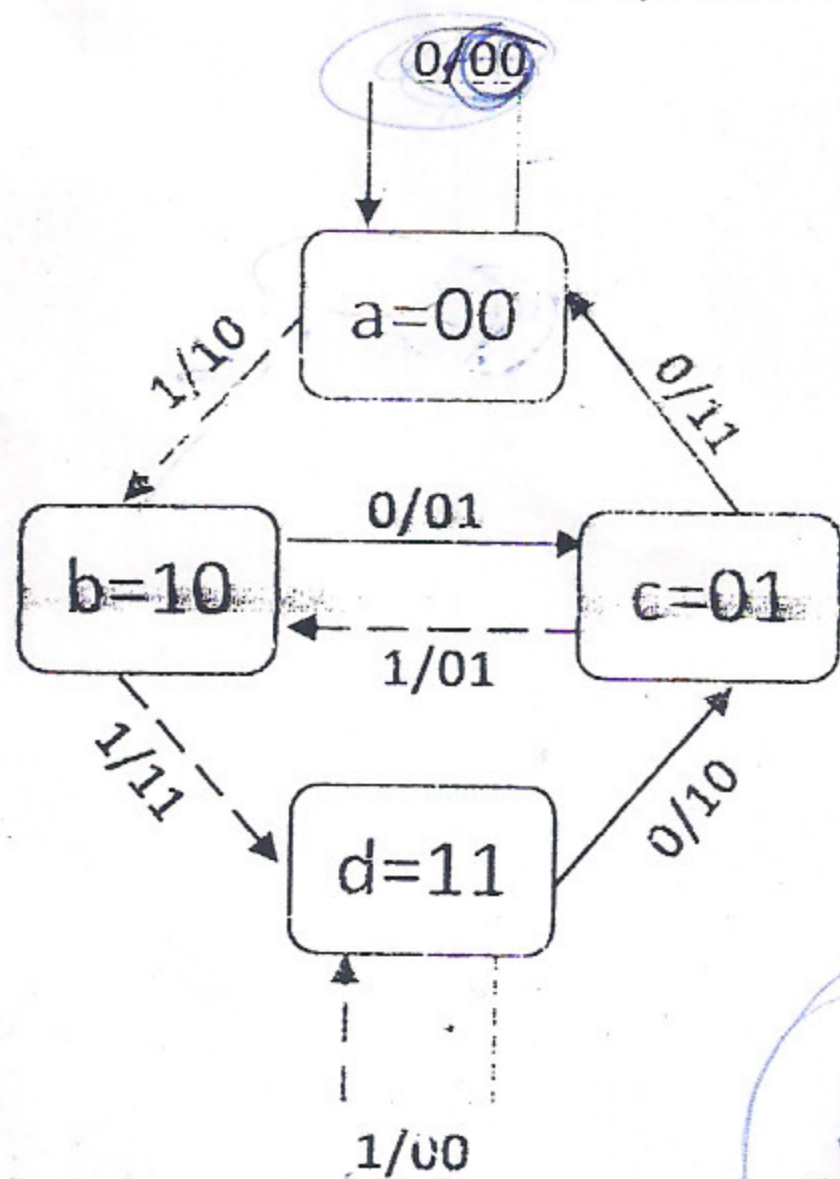
Q2: (20 points) Consider a (7,4) linear block code with generator matrix G

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 7$$

- Find all the codewords of the code.
- Find the **parity-check matrix** H .
- Find **code bits**, **message bits**, **parity bits**, **code rate**, **minimum distance**, **error detection capability** and **error correction capability**.
- Compute the **syndrome** for the received vector 1101101. Is this a valid code vector?

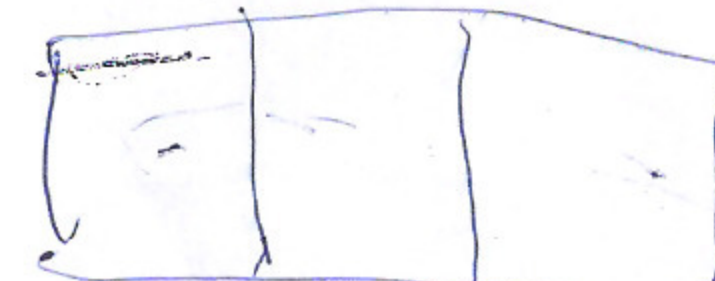
Q3: (15 points) For the following state diagram.

- Draw the **encoder** using shift registers, modulo-2 adders, and a commutator?
- Find the **generator polynomial** of the encoder $g(x)$.
- Encode the input sequence bits $m = 10101$?



$$S = VH^T =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$g_1(x) = 1 + x^2$$

$$g_2(x) = x + x^2$$

Good luck

الفصل الدراسي: خريف 2015-2016 اسم الاستاذ/المنسق: حسام الدين الهشيري. رقم القيد: المجموعة:

Q1: In a telegraph source having two independent symbols dot and dash, the dot duration is 0.1s and the dash duration is 0.5s. The probability of dot occurrence is four times that of dash and the time separation between symbols is 0.1s. Find the information rate of this telegraph source? (16 Marks)

Q2: Consider a DMS S with symbols $s_i, i=1,2,3,4,5,6$ and with probabilities $p(s_1)=0.36, p(s_2)=0.24, p(s_3)=0.15, p(s_4)=0.12, p(s_5)=0.08,$ and $p(s_6)=0.05$. Construct a Huffman code for the source and find the efficiency of the constructed code? (16 Marks)

Q3: Consider a systematic block code whose parity check equations are:

$$\begin{aligned} b_0 &= m_0 + m_1 + m_3 \\ b_1 &= m_0 + m_2 + m_3 \\ b_2 &= m_0 + m_1 + m_2 \\ b_3 &= m_1 + m_2 + m_3 \end{aligned}$$

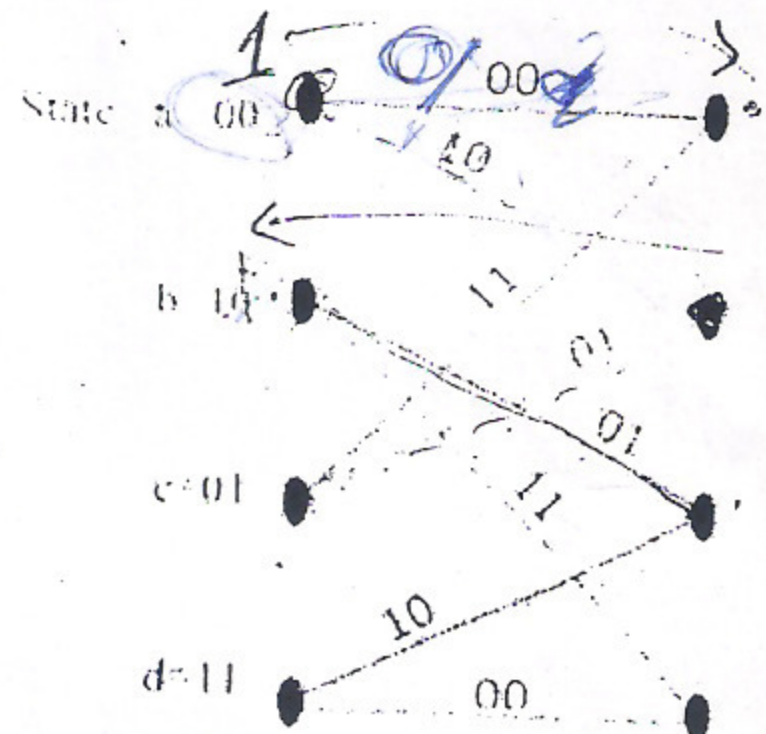
Where m_i are the message bits, $i=0, 1, 2, 3$, and b_i are the check bits, $i=0, 1, 2, 3$.

- (a) What are the parameters n and k ? Find the generator matrix for the code. (2 Marks)
- (b) What is the minimum Hamming distance? How many errors can the code correct? (2 Marks)
- (c) Is the vector $[1 0 1 0 1 0 1 0]$ a valid codeword? (2 Marks)
- (d) Is the vector $[0 1 0 1 1 1 0 0]$ a valid codeword? (2 Marks)

$$e = d_{min} - 1 \quad t = \lfloor \frac{d_{min} - 1}{2} \rfloor$$

Q4: For the given encoder trellis diagram:

- a) Draw the encoder state diagram? (4 Marks)
- b) Draw the encoder? (show your answer) (4 Marks)
- c) Write the generator polynomial of the encoder? (2 Marks)
- d) Encode the message sequence $[01101]$. (5 Marks)

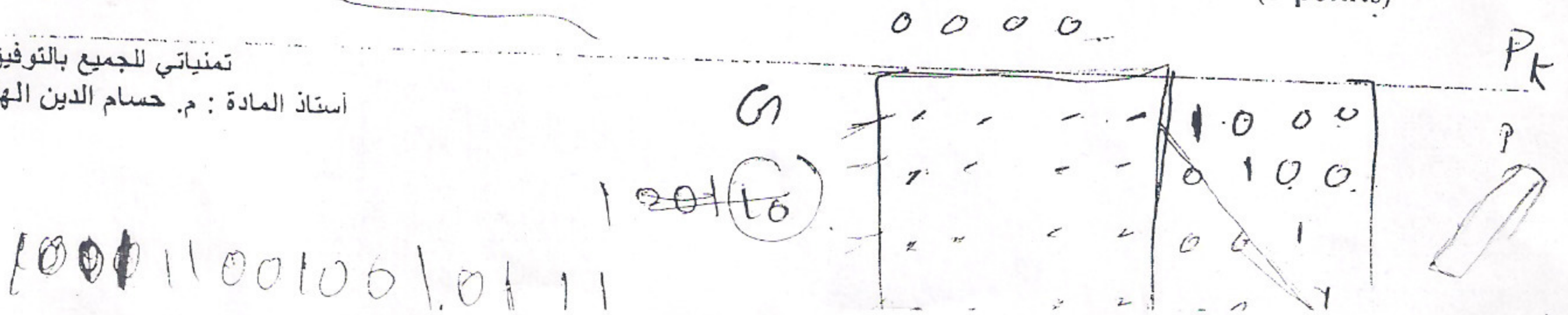


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Q5: Given that the mutual information $I(X; Y) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log(p(y/x)/p(y))$. derive the follow $I(X; Y) = H(X) - H(X/Y)$? (5 points)



تمنياتي للجميع بالتوفيق
 استاذ المادة: م. حسام الدين الهشيري



Information Theory and Coding (CM303)

Midterm Exam (40%)

07 June, 2015

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1. What is the maximum entropy $H(s)$ for Binary Discrete Memoryless Source (BDMS)? Prove your answer!

(10 points)

$$\frac{2}{3} \times 0.6 = 0.4$$

$$T_d = 0.6 \text{ sec}$$

$$T = 0.55$$

Q2. A telegraph source having two symbols, dot and dash. The dash duration is 0.6 seconds; and the dot duration is two third of the dash duration. The probability of the dot occurring is twice that of the dash, and the time between symbols is 0.2 seconds. Calculate the information rate of the telegraph source?

(10 points)

occurrence
SDM
RB

Q3. A discrete memoryless source has an alphabet of seven symbols with probabilities for its output as described in following table:

Symbol	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

1. Construct a Shannon-Fano code for the source and calculate the efficiency of coding?

(10 points)

2. Construct a Huffman code for the source and calculate the efficiency of coding? And compare the results?

(10 points)

Handwritten notes and scribbles on the left margin.

Good luck!

Information Theory and Coding (CM303)

Answer of Midterm Exam (40%)

07 June, 2015

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Time Allowed: 2 hours

Q1. Answer:

Since it BDMS, then the source has two symbols s_1 and s_2 . Let the probability of s_1 is $p(s_1)=a$ then the probability of s_2 is $p(s_2)=1-a$. The entropy $H(s)$ of this source:

$$H(s) = -a \log_2(a) - (1-a) \log_2(1-a)$$

Note that when $a=0 \rightarrow H(s)=0$

$$a=1 \rightarrow H(s)=0$$

The maximum entropy can be found by the differentiation of $H(s)$:

$$\frac{dH(s)}{da} = \frac{d(-a \log_2(a) - (1-a) \log_2(1-a))}{da} = -\left[a \cdot \frac{1}{a} - \log_2(a)\right] - \left[1-a \cdot \frac{1}{1-a} - \log_2(1-a)\right]$$

$$\frac{dH(s)}{da} = -\log_2(a) + \log_2(1-a)$$

$$\frac{dH(s)}{da} = \log_2\left(\frac{1-a}{a}\right)$$

The maximum is found when $\frac{dH(s)}{da} = 0$

$$\log_2\left(\frac{1-a}{a}\right) = 0 \text{ when } \frac{1-a}{a} = 1$$

$$\therefore a=0.5$$

Which means when $a=0.5$ $H(s)$ is maximum

$$H(s) = -0.5 \log_2(0.5) - (1-0.5) \log_2(1-0.5) = 1 \text{ bit/symbol}$$

Q2. Answer:

- Given that:
1. Dash duration: 0.6 sec.
 2. Dot duration: $2/3 \times 0.6 = 0.4$ sec.
 3. $P(\text{dot}) = 2 P(\text{dash})$.
 4. Space between symbols is 0.2 sec.
- Information rate = ?

1. Probabilities of dots and dashes:

Let the probability of a dash be "P". Therefore the probability of a dot will be "2P". The total probability of transmitting dots and dashes is equal to 1.

$$\therefore P(\text{dot}) + P(\text{dash}) = 1$$

$$\therefore P + 2P = 1 \quad \therefore P = 1/3$$

$$\therefore \text{Probability of dash} = 1/3$$

$$\text{And Probability of dot} = 2/3$$

2. Average information $H(X)$ per symbol:

$$\therefore H(X) = P(\text{dot}) \cdot \log_2 [1/P(\text{dot})] + P(\text{dash}) \cdot \log_2 [1/P(\text{dash})]$$

$$\therefore H(X) = (2/3) \log_2 [3/2] + (1/3) \log_2 [3] = 0.3899 + 0.5283 = 0.9182 \text{ bits/symbol.}$$

3. Symbol rate (Number of symbols/sec.):

The total average time per symbol can be calculated as follows:

$$\text{Average symbol time } T_s = [T_{\text{DOT}} \times P(\text{DOT})] + [T_{\text{DASH}} \times P(\text{DASH})] + T_{\text{space}}$$
$$\therefore T_s = [0.4 \times 2/3] + [0.6 \times 1/3] + 0.2 = 0.6667 \text{ sec/symbol.}$$

Hence the average rate of symbol transmission is given by:

$$R_s = 1/T_s = 1,5000 \text{ symbols/sec.}$$

4. Information rate (R_I):

$$R_I = R_s \times H(s) = 1,5000 \times 0.9182 = 1,3773 \text{ bits/sec.}$$

1,3773

Q3. Answer:

1. Shannon-Fano code:

Symbols	Probability	Step 1	Step 2	Step 3	Step 4	Code word
S_0	0.25	0	0			00
S_1	0.25	0	1			01
S_2	0.125	1	0	0		100
S_3	0.125	1	0	1		101
S_4	0.125	1	1	0		110
S_5	0.0625	1	1	1	0	1110
S_6	0.0625	1	1	1	1	1111

Average code word length (L):

$$L = \sum_{k=0}^6 p_i \times n_i$$

$$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$$

$$= 2.6250 \text{ bits/message}$$

Entropy of the source (H):

$$H(s_i) = \sum_{k=0}^6 p_i \times \log_2(1/p_i)$$

$$= 0.25 \log_2(1/0.25) + 0.25 \log_2(1/0.25) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.0625 \log_2(1/0.0625) + 0.0625 \log_2(1/0.0625) = 2.6250 \text{ bits/symbols}$$

$$\text{Code efficiency } \eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$$

$$\therefore \eta = 100\%$$



الاسم: رقم القيد:

Q1. (10 Marks) In a TV transmission, picture consists of 2×10^6 elements, 32 different brightness levels and pictures are repeated at a rate of 32 pictures per second. If the brightness levels have equal likelihood of occurrence and picture elements are independent, find average information rate of this TV source?

Q2. (12 Marks, 3 each) For the Binary Symmetrical Channels shown below:

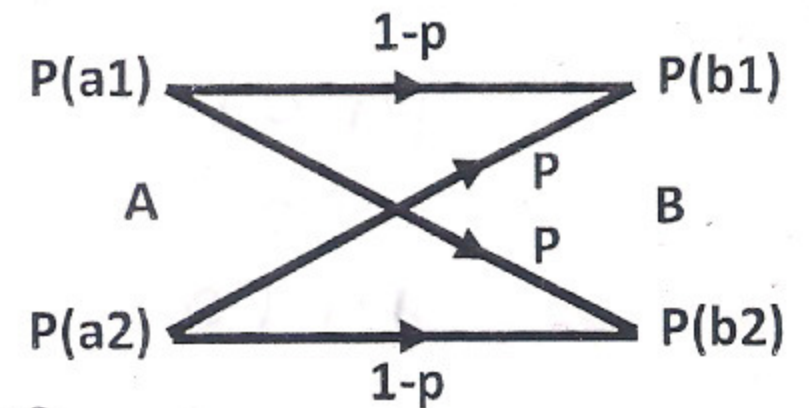
a) Find the Channel capacity when $p = 1$, $p = 0$, $p = 0.5$, and $p = 0.3$?

b) Find the Maximum Capacity of the Channel?

c) Find the Input, output, channels forward probabilities, $H(A)$, ...

$H(B)$, and $H(B/A)$ when the channel works at Maximum Capacity?

d) How can you make the channel works at half of its Maximum Capacity?



Q3. (8 Marks, 2 each) Consider a DMS Source with symbols $S_i, i=1,2,3,4$. Table below lists 6 possible binary codes

a) Find which of them distinct codes are?

b) Find which of them prefix-free codes are?

c) Find whether instantaneous codes exist for these codes?

d) Can you decide which code is the best code for this source, and why?

Table of Codes of the source S						
S_i	C 1	C 2	C 3	C 4	C 5	C 6
S1	00	11	01	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

Q4. (15 Marks, 3 each) Consider a Systematic Linear Block Code whose parity check equations are:

$$P_0 = m_0 + m_1 + m_3, P_1 = m_0 + m_2 + m_3, P_2 = m_0 + m_1 + m_2, \text{ and } P_3 = m_1 + m_2 + m_3,$$

Where m_i are the message bits, $i = 0, 1, 2, 3$, and P_i are the check bits, $i = 0, 1, 2, 3$.

(a) Find the generator matrix of the code and draw the Encoder?

(b) Find code bits, message bits, parity bits, code rate, Hamming weight?

(c) Find Hamming distance, the error-detection and error-correction capabilities of the code?

(d) Find the syndrome look-up table?

(e) Are the vectors $[1 0 1 0 1 0 1 0]$ and $[0 1 0 1 1 1 0 0]$ valid codewords? (show the answer steps)

Q5. (4 Marks) Given a Binary Convolutional Encoder with $K=3$, rate $1/3$, and Impulse Response 101011010. Encode the input sequence bits $m = 10101$. And write down the polynomial equations of the encoder?

Q6. (3 Marks) Consider a (4,1,4) convolutional encoder with the following generator polynomials:

$g_1 = [1010], g_2 = [0101], g_3 = [1110], g_4 = [1001]$. Draw the encoder and how many states does this encoder have?

Q7. (8 Marks, 2 each) Briefly answer the following:

[1] How do we measure information content in a message?

[2] What reduces mutual information between input and output of a channel?

[3] What is the purpose of source coding and channel coding?

[4] What are the advantages of convolutional codes over block codes?